

## Note on the PROMETHEE net flow computation

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28/7/2015

### Introduction

This note describes special cases for the computation of the PROMETHEE net flow. In each case we consider a multicriteria decision problem including a set of  $n$  actions:

$$A = \{a_1, a_2, \dots, a_i, \dots, a_n\} \quad (1)$$

and  $k$  criteria that have to be maximized:

$$f_1, f_2, \dots, f_j, \dots, f_k \quad (2)$$

The weights  $w_j$  of the criteria are normalized in the following way:

$$\sum_{j=1}^k w_j = 1 \quad (3)$$

and the preference functions associated to the criteria are noted as follows:

$$P_j(\cdot, \cdot) \quad (4)$$

The unicriterion net flow for criterion  $f_j$  is then defined as:

$$\phi_j(a) = \frac{1}{n-1} \sum_{b \neq a} [P_j(a, b) - P_j(b, a)] \quad (5)$$

And the multicriteria net flow is the weighted sum of the unicriterion net flows:

$$\phi(a) = \sum_{j=1}^k w_j \phi_j(a) \quad (6)$$

### Type 3 – V-shape preference function

Let us consider that a V-shape preference function is associated to criterion  $f_j$  and that the preference threshold  $p_j$  is such that:

$$p_j \geq \max_{a \in A} f_j(a) - \min_{a \in A} f_j(a) \quad (7)$$

For instance, for evaluations between 0 and 1,  $p_j$  should be larger than or equal to 1. In that case the preference function is linear (for positive deviations) and thus:

$$P_j(a, b) - P_j(b, a) = \frac{1}{p_j} [f_j(a) - f_j(b)] \quad (8)$$

The unicriterion net flow becomes:

$$\begin{aligned}
\phi_j(a) &= \frac{1}{n-1} \sum_{b \neq a} \frac{1}{p_j} [f_j(a) - f_j(b)] \\
&= \frac{1}{n-1} (n-1) \frac{1}{p_j} f_j(a) - \frac{1}{n-1} \frac{1}{p_j} \sum_{b \neq a} f_j(b) \\
&= \frac{1}{p_j} f_j(a) - \frac{1}{n-1} \frac{1}{p_j} [n \bar{f}_j - f_j(a)] \\
&= \frac{n}{n-1} \frac{1}{p_j} [f_j(a) - \bar{f}_j]
\end{aligned} \tag{9}$$

where  $\bar{f}_j$  is the arithmetic average of the evaluations.

The unicriterion net flow value is thus a linear function of the evaluation.

The multicriteria net flow also is a linear function of a weighted sum of the evaluations:

$$\phi(a) = \frac{n}{n-1} \left[ \sum_{j=1}^k \frac{w_j}{p_j} f_j(a) - \sum_{j=1}^k \frac{w_j}{p_j} \bar{f}_j \right] \tag{10}$$

When all the criteria have the same preference threshold  $p$  the multicriteria net flow is a linear function of the weighted sum of the evaluations with weights  $w_j$  and PROMETHEE II is equivalent to the weighted sum method:

$$\phi(a) = \frac{n}{n-1} \frac{1}{p} \left[ \sum_{j=1}^k w_j f_j(a) - \sum_{j=1}^k w_j \bar{f}_j \right] \tag{11}$$

### Type 1 – Usual criterion

For a usual criterion, the preference function is such that:

$$P_j(a,b) - P_j(b,a) = I_{f_j(a) > f_j(b)} - I_{f_j(a) < f_j(b)} \tag{12}$$

where  $I$  are indicator (binary) variables. Thus:

$$\sum_{b \neq a} [P_j(a,b) - P_j(b,a)] = \left| \left\{ b \mid f_j(a) > f_j(b) \right\} \right| - \left| \left\{ b \mid f_j(a) < f_j(b) \right\} \right| \tag{13}$$

and

$$\phi_j(a) = \frac{1}{n-1} \left[ \left| \left\{ b \mid f_j(a) > f_j(b) \right\} \right| - \left| \left\{ b \mid f_j(a) < f_j(b) \right\} \right| \right] \tag{14}$$

From the definition of the average rank it comes that:

$$\phi_j(a) = \frac{n+1}{n-1} - \frac{2}{n-1} R_j(a) \tag{15}$$

where  $R_j(a)$  is the average rank of  $f_j(a)$  in the set of the evaluations for criterion  $f_j$ .

When all the criteria are associated to usual preference functions, the multicriteria net flow thus becomes:

$$\phi(a) = \frac{n+1}{n-1} - \frac{2}{n-1} \sum_{j=1}^k w_j R_j(a) \quad (16)$$

In this case the PROMETHEE II ranking is equivalent to the Borda method.